Specifying and Analyzing Transactional Consistency Models with Predicates (Extended Abstract)

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Motivation. Database transactions use predicate operations (e.g., SELECT ... WHERE) to access a set of items matching a condition. Under concurrency, such operations may expose predicate anomalies [3], that a consistency model must specify as admissible or forbidden. Yet consistency model theories that account for predicates remain inadequate:

(1) Specification frameworks. The Adya lineage [1, 2, 8–10, 15] reason via dependency graphs, yet these accounts stay informal (e.g., the key notion of *changing matches* is vague) and vary across papers (e.g., *predicate-read-dependency* are defined differently in [1] and [2]). Cerone *et al.*'s framework [6, 7] offers declarative axioms for consistency models but misses predicates.

(2) Characterization theorems. Serializable (SER) histories without predicates can be characterized by acyclic dependency graphs [4]. The Adya lineage incorporates predicate edges into such graphs, but, surprisingly, none of them establishes a correctness argument, appealing instead to the classic *item-only* SER characterization theorem [4, 12]. The graph characterization of snapshot isolation (SI) in [7] is likewise predicate-free. To our knowledge, no prior work gives an SI characterization for histories with predicates.

Contributions. We address these gaps as follows: (1) We present an axiomatic specification framework for consistency models with predicates, serving as a rigorous correctness criterion. (2) We provide the first formal definition of dependency graphs constructed from histories with predicates. We then propose the SER and SI characterization theorems in terms of dependency graphs, and prove them with respect to the axiomatic framework. Our theory provides rigorous foundations for consistency models with predicates, enabling history checking [5, 13, 14, 16], protocol verification [11], and robustness analysis [7, 10] in the presence of predicates.

Approach. First, we extend the execution model [6] with predicate reads (predicate writes are predicate reads followed by writes on matched items). To this end, we classify each predicate read in a transaction *item-wise* as internal or external: the read is *external* on item x if it is the first predicate read covering x in the transaction, and there is no preceding item write on x before it. The key Ext-Pred axiom requires that in an execution $\mathcal{X}$, an external predicate read r in transaction T on a covered item x is explained by the last transaction T' in the commit order among those visible to the read r that write x. This is captured by the predicate-read-dependency relation $T' \xrightarrow{\text{Pred-WR}_{\mathcal{X}}(x)} T$: (1) if x is included in the result set, the returned (x, v) pair is written by T' and the value v satisfies the predicate P; (2) if excluded, T''s visible value on x cannot satisfy P.

Second, we construct dependency graphs from *histories* with predicates. Specifically, we introduce Pred-WR and Pred-RW edges. A Pred-WR edge $S \xrightarrow{\text{Pred-WR}(x)} T$ means that S's write on x is the version observed by T's external predicate read on x. A Pred-RW edge $T \xrightarrow{\text{Pred-RW}(x)} T'$ arises if T' overwrites x that *changes matches* (described later) of T's external predicate read on x.

Third, we propose the SER and SI characterization theorems. Specifically, we show that a history satisfies SER iff it admits an acyclic dependency graph (including Pred-WR and Pred-RW), while a history satisfies SI iff it admits a graph where every cycle contains two adjacent anti-dependency edges (RW or Pred-RW), generalizing [7, Theorem 4.1]. [1]

Challenges. We identify three challenges in developing our theory, regarding the subtle semantics of predicate reads.

(1) Changing matches. Existing works are ambiguous about the key notion of *changing matches*. We treat it rigorously: Transaction S *changes matches* of T's external predicate read on x with respect to T', denoted $\Delta(T', T, S, x)$, if the (x, v') pair installed by T' matches P of this predicate read whereas the (x, v_s) pair installed by S does not or vice-versa, or both (x, v') and (x, v_s) match P but with different values $v' \neq v_s$.

(2) Justifying Ext-Pred. A crucial step in proving both theorems is to justify Ext-Pred for external predicate reads, and the difficulty lies in deriving Pred-RW edges to conclude contradiction. For transaction T with external predicate read on item x, assume by contradiction that $S \xrightarrow{\text{Pred-WR}(x)} T$ does *not* hold, where S is the last transaction visible to T on x, and that $T' \xrightarrow{\text{Pred-WR}(x)} T$ holds for $T' \neq S$. Then, Pred-WR forces values on S and T' that satisfy $\Delta(T', T, S, x)$, leading to desired $T \xrightarrow{\text{Pred-RW}(x)} S$.

(3) Weakening graph-execution correspondence. The "if" direction of the SI theorem follows the incremental construction of (partial) executions $\mathcal{P}$ satisfying SI from dependency graphs $\mathcal{G}$ in [7, Theorem 4.1], which relies on the graph-execution correspondence lemma in [7, Lemma 4.7]. However, this lemma is too strong with predicates: for an item x covered but excluded from the result set, several transactions may write non-matching values. The graph-level witness picked by Pred-WR(x) in $\mathcal{G}$ may *not* coincide with the last visible writer specified by Pred-WR$_{\mathcal{P}}(x)$ in $\mathcal{P}$. Therefore, we prove a weaker correspondence sufficient for SI.

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[1] Efficient graph construction is orthogonal to the characterization results and is left for future work.

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